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The Structure and Growth of International Trade

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Abstract

We use a model of proportionate growth to describe the dynamics of international trade flows. We provide an explanation to the fact that the extensive margin of trade account for a large fraction of the greater exports of large economies, as well as for a number of stylized facts described by the literature on trade networks such as the power-law distribution of connectivity and the fat tails displayed by the distribution of the growth rates of trade flows. Hence, such a simple setup is able to capture the dynamics of very different economic variables, from firm size (as shown in the industrial organization literature) to international trade flows (as documented here). Furthermore, we provide an additional element to the discussion on the relative ability of different international trade models to adequately match empirical regularities.

JEL Codes: F31, F33, F42.
Keywords: international trade, growth, network analysis

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1 Introduction

We present a simple stochastic model of proportionate growth to describe international trade flows as a set of transactions of different magnitude occurring among countries, and we test it using both simulations and real data.

This very simple setup – that enjoys a long tradition in the industrial organization literature (Simon 1955, Ijiri and Simon 1977, Sutton 1997) and has recently been applied to the study of many real-world networks by Barabsi and Albert (1999) – allows us to discuss many interesting empirical findings recently highlighted by international economists (Hummels and Klenow 2005, Fagiolo et al. 2009). In particular, the model accounts for the prominent role played by the extensive margin of trade in explaining a large fraction of the greater exports of large economies. Moreover, it is able to replicate other structural properties of international trade such as the coexistence of a bulk of small-valued trade relationships with a small number of very strong commercial links.

The main contribution of the paper is twofold. First and foremost we add to the discussion triggered by the Hummels and Klenow (2005) on the relative merits of various international trade models to adequately account for empirical facts. Indeed, we show that a simple model of preferential attachment is able to capture relevant features of the data and we therefore provide a stochastic benchmark against which theories can be tested. Future international trade models should then focus on *departures* from this stochastic benchmark, since they represent the true outcome of economic forces determining trade patters. Second, we provide further evidence that a simple stochastic model is able to capture the dynamics of a large family of economic phenomena at different levels of aggregation, from business units, industrial sectors and country GDP (Fu et al. 2005) to international trade flows (as we show here).

The paper is organized as follows: Section 2 briefly discusses the relevant existing literature, while Section 3 presents the model and its most important implications. Next we present the data and show that a number of stylized facts of international trade are consistent with the predictions of the model. Section 5 introduces the simulation and discusses its main results. We then lie down some conclusions and outline possible patterns for future research.

2 Background literature

Recent developments in the theory of international trade, together with the increasing availability of large-scale collections of (product- and firm-level) microdata have lead economists to investigate the micro foundations of international trade dynamics and their key driving forces. This new wave of research has pro-

duced a number of stylized facts, some of which have already been accounted for by models, while other calls for further theoretical investigation.

In particular, empirical work has been putting a lot of attention in understanding the role that extensive and intensive margins play in determining total export. The theoretical revolution triggered by Melitz (2003) explicitly calls for this kind of disaggregation by making heterogeneity among exporters one of the key ingredients of international trade models. In such a context in fact, international trade induces the more productive firms to enter the export market and simultaneously forces the least productive firms to exit, thus modifying aggregate trade flows along both the extensive (how many firms export) and the intensive (how much is exported by each firm) margin.

In this framework Chaney (2008) looks at the gravity structure of bilateral trade flows once heterogeneity among exporters is accounted for. He shows that the elasticity of substitution has opposite effects on each margin: higher substitutability makes the intensive margin more sensitive to changes in trade costs, whereas the extensive margin becomes less sensitive so them. Crozet and Koenig (2008) bring the model to the data using information on French exporting firms and find that for a large majority of industrial sectors, the estimated parameters are consistent in size and sign with theory.

Besedes and Prusa (2006) and Besedes and Prusa (2007) explore extensively the respective roles of extensive and intensive margins in determining export growth. The distinctive feature of the papers is to show that the vast majority of bilateral trade relations (at a fine level of product disaggregation) dies very young, the median survival time being 1 or 2 years. Hence, the establishment of new export relations is of very minor importance for long-term export dynamics, whereas the bulk of trade growth depends on the survival and deepening of existing relations.

In a similar vein, an influential paper by Hummels and Klenow (2005) investigates the influence of the extensive and intensive margins in explaining the simple observation that large economies trade more in absolute terms. They report that the extensive margin accounts for nearly 60 percent of the greater export of larger economies, while within the same product category richer countries export higher quantities at slightly higher prices. The authors then compare these stylized facts with the predictions issued by a set of workhorse trade models, and show that different features of the models match some of the facts, but we still lack a setup capable to effectively account for all the empirical findings.

An even larger set of stylized facts about international trade has emerged from the growing body of literature that studies trade using complex network analysis. While the idea of representing the web of trade relationships as a network is not new (Snyder and Kick 1979, Breiger 1981), more recent works find their root in the (econo)physic community, where world trade is just another example

of an interesting complex network. Early works analyzed binary versions of the international trade network (ITN), considering a link between two countries as present whenever a positive trade flow between them takes place and disregarding the heterogeneity among relationships (Serrano and Boguñá 2003, Garlaschelli and Loffredo 2004, Garlaschelli and Loffredo 2005). More recently however, the focus of research has shifted toward weighted versions of the graph, acknowledging that the strong heterogeneity existing among links provides additional information that cannot be overlooked (Bhattacharya et al. 2008, Fagiolo et al. 2008, Fagiolo et al. 2009).

As mentioned above, this literature has produced a large pool of stylized facts that, at least for what concerns the ITN, are still waiting for a theoretical explanation. In particular, empirical analysis confirms the existence of a scale-free degree distribution implying high heterogeneity among the role played by countries in the network. There is also evidence of a hierarchical structure in the ITN (Snyder and Kick 1979, Breiger 1981) since the network displays negative assortativity, and the clustering coefficient depends negatively on node degree. Out of network jargon this means that countries with few connections tend to link to highly-connected hubs. Moreover, all studies find that the key properties of the ITN are remarkably stable over of time (Garlaschelli and Loffredo 2005, Fagiolo et al. 2009).

Fagiolo et al. (2008) note that the statistical properties of the weighted version of the network differ substantially from those exhibited by its binary counterpart. The weighted network is weakly disassortative, and well-connected countries tend to trade with partners that are strongly linked between themselves. Finally, the distribution of node strength is very skewed to the right suggesting that a few intense connections coexist with a majority of very weak ones: this is consistent with Bhattacharya et al. (2008), who approximate the distribution of positive link weights by means of a log-normal density.

The most complete attempt to characterized the structural properties of the ITN is probably represented by Fagiolo et al. (2009). They analyze the statistical properties of both (positive) link weights and node characteristics (such as connectivity, assortativity, clustering and centrality), provide an in-depth analysis of the network topology, and fix a number of relevant stylized facts, which confirm and expand the list established by previous authors. Among the most important regularities, in the present context we remember (i) the log-normal distribution of link weights, (ii) the heavy tails displayed by the growth rates of link weights, which are not log-normally distributed and call for a model of preferential attachment; (iii) the negative assortativity of the network, which is very pronounced for the binary network, milder for its weighted counterpart.

Finally, it is worth noting that an increasing share of the economic profession is recognizing the contribution that network analysis can give to understanding eco-

nomic dynamics, as is testified by recent works by Hidalgo et al. (2007), Hidalgo and Hausmann (2009), Kali and Reyes (2009). The first article studies development as a process of transformation whereby economies acquire the ability to produce and export more complex (i.e. higher value added) products. The product space is modeled as a network and the probability of developing a new product is inversely related to its distance from the country's existing production. Hidalgo et al. (2007) show that industrial countries populate the dense core of the product space, whereas developing nations tend to be specialized in commodities that are more peripheral, this making it difficult for them to climb up the quality ladder. Hidalgo and Hausmann (2009) continue along this *network view of economic development* by analyzing a tripartite graph linking countries to the products they export and the capabilities needed to produce them. The authors claim that complexity of a country's economic structure is well-proxied by the characteristics of the products it exports and is a good predictor of GDP per capita. Last, Kali and Reyes (2009) focus on the role of the international trade network in channeling shock transmission and contagion. In particular they show empirically that a crisis is amplified if the epicenter country is better integrated into the trade network. However, target countries affected by such a shock are in turn better able to dissipate the impact if they are well integrated into the network.

3 The model

We model international trade as a set of transactions of different magnitude occurring among countries. Our setup consists of a reformulation of a fairly old idea that goes back to Simon (1955), and has been extensively used to model the dynamic of socio-economic systems between the 1950s and 1970s. Recently, a version of the model has been applied by Barabsi and Albert (1999) to network dynamics, showing that this simple setup is able to account for many of the stylized facts observed in real-world networks. The increasing interest in the study of *weighted* versions of networks calls for an extension of the original Simon-Barabasi model capable of accounting for the large degree of heterogeneity across link values. The easiest (and less demanding in terms of assumptions) way to implement this is to assume that the magnitude of links grows according to the so-called Gibrat's *law of proportionate effects*, which postulates that the expected value of the growth rate of a business firm is independent of its current size. In recent years, generalization of this idea have been used to rationalize the stylized fact that the distribution of the growth rates of economic organizations ranging from company divisions up to country GDPs (Growiec et al. 2008).

We therefore propose a slightly modified version of the Simon-Barabasi model to describe the dynamic and growth of international trade relations. Two key

assumptions in the model are the following (Simon 1955, Sutton 1997, Barabsi and Albert 1999):

1. each country i takes part into $K_i(t)$ transactions. At time $t = 0$ there are N_0 countries linked by n_0 relationships. At each time step t, a new trading opportunity among two countries arises: thus the number of transactions occurring at time t is $n_t = n_0 + t$;

2. with probability a the new trading opportunity is assigned to a new country, whereas with probability $1-a$ it is allocated to an existing country i. In this latter case, each country captures trading opportunities with a probability p_i that is proportional to the number of relationships already established: $p_i = (1-a)\frac{K_i(t)}{n_t}$. Hence, at each time t this rule identifies a pair of (distinct) countries that establish a new trade relationship.

Moreover, we assume that each trade relationship grows in time according to an independent random process.

3. at time t link l between countries i and j has size $w_{ij}^l(t) > 0$, where K_i, K_j and $w_{ij}^l(t) > 0$ are independent random variables. At time $t+1$ the size of each exchange is increased or decreased by a random factor $x_{ij}^l(t)$, so that $w_{ij}^l(t+1) = w_{ij}^l(t)x_{ij}^l(t)$. The shocks $x_{ij}^l(t)$ are taken from a distribution with finite mean (μ_x) and standard deviation (σ_x).

Based on the first set of assumptions we derive the connectivity distribution as in Barabsi and Albert (1999) and Buldyrev et al. (2007). In the limit of large t when $a = 0$ (no entry), the distribution of K_i converges to an exponential; on the contrary when $a > 0$ (and small) the connectivity distribution at large t converges to a power-law with an exponential cutoff.

Fu et al. (2005) find an analytical solution for the distribution of the growth rates of the size of trade relationships for the case when $t \to \infty$ and $a \to 0$, showing that it combines a double exponential (Laplace) body with a power law decay in the tails.

A further implication of the model can be derived from the third assumption and concerns the distribution of the magnitude of trade relationships. First, since the multiplicative process evoked in assumption (3) follows Gibrat's law, the size of trade relationship is log-normally distributed. Second, Growiec et al. (2008) show that upon aggregation the log-normal distribution is multiplied by a stretching factor that can lead to a Pareto upper tail: in our context this applies to total trade of each country in the network.

Moreover, a negative relationship exits among the value of each trade link and the variance of its growth rate. Riccaboni et al. (2008) illustrate how simple models of proportionate growth imply an approximate power-law behavior for the

variance of growth rates of the form $\sigma = w^{-\beta(w)}$ where $\beta(w)$ is an exponent that weakly depends on the size of the trade relationships w_{ij}^l. In particular, $\beta = 0$ for small values of w_{ij}^l, $\beta = 0.5$ for $w_{ij}^l \to \infty$, and it is well approximated by $\beta \approx 0.2$ for a wide range of intermediate values of w_{ij}^l.

Last, the model yields a prediction also on the relation between the number of trading relationships established by a country and their average size. In fact, since the size of each transaction is sampled from a log-normal distribution (i.e. since the w_{ij}^l are log-normally distributed), and given the skewness of such a density function, most of the draws will be concentrated in the lower tail of the distribution. In other words, the probability to draw a large value for a transaction increases with the number of draws, thus generating a positive correlation between K_i and the average link size $\bar{w}_{ij}$.[1] Using the number of products traded by each country as a proxy for K_i, we can interpret this relationship as one between the extensive and the intensive margins of trade. Hence, since total export is just the product of the number of transactions by their average size, we end up with a relationship echoing the main finding by Hummels and Klenow (2005), namely that the extensive margin accounts for a large share of the greater exports of large economies.

4 Empirical evidence

4.1 Data

We use the NBER-United Nations Trade Data documented by Feenstra et al. (2005) and available through the Center for International Data at UC Davis to assess the ability of the model to replicate the main features of the WTN. This source provides bilateral trade flows among a large number of countries over 1962–2000, both aggregated and at 4-digit SITC level. Data are in thousands US dollars and, for product-level flows, there is a lower threshold at $ 100,000 below which transactions are not recorded. One point to note is that microdata are not always consistent with country trade flows: in a number of cases we do not observe any 4-digit transaction recorded between two countries, but nevertheless find a positive total trade, and vice-versa. Since we take the number of product traded among any two country pairs as the empirical counterpart of the number of transactions

[1] Another way to think about this issue is in terms of convergence to the central limit theorem. Skewness of the underlying distribution causes convergence to normality to be slow: hence, repeated draws from a lognormal distribution will not converge to normality unless the number of draws is very large. Normality would imply no relation between the number of links and their average value, whereas departures from it (i.e. a slow convergence) determine a positive correlation between the two variables since a vast majority of trade relationships will has very small size due to the concentration of probability on the lower tail.

K_i, to avoid inconsistency between micro- and macro-data we compute the total trade by aggregating product-level data.

In what follow we only consider trade data for the period between 1992–2000, in order to minimize the effects induced by the variation in the number of countries due to geopolitical events such as the breaking up of Yugoslavia and the Soviet Union. Moreover, we drop a number of small economies (e.g. Gibraltar or Guadeloupe) for which trade data exists but are not exhaustive; we also aggregate information for some countries (e.g. the Czech Republic and Slovakia) to keep the number of economies constant over time.[2] In this way we end up with a balanced panel of 166 countries.

4.2 Stylized facts

The first use of the data in our work is to compare the implications of the model with empirical observation. Later, we will use information in the data also to calibrate the simulations and check for the ability of the model to replicate real-work phenomena by comparing simulated and actual trade flows. We know from previous work (Fagiolo et al. 2009) that the main features of the ITN are broadly consistent with the model we propose. Here we look in more details at some specific characteristics of international trade flows.

Figure 1 shows that the number of 4-digit SITC products traded (the empirical counterpart to the trading opportunities described in the model) is Pareto with an exponential cutoff. The main plot displays the probability distribution in log-log scale, whereby the power-law is the straight line body, and the exponential cutoff is represented by the right tail. The inset presents the same phenomenon in semi-log scale: this time it is the exponential part of the distribution that becomes a straight line, so that with this trick we can magnify what happens to the probability distribution as K_i grows large. As we have seen in Section 3 above, the presence of an exponential cutoff suggests the existence of moderate entry of new players in the networks: empirically this is represented by the countries emerging from the collapse of the Soviet Union and former Yugoslavia that, though not starting from scratch, had nonetheless to rebuild their network of trade relationships from low levels of connectivity.[3]

Moving to the weighted version of the network, one can look at the distribution of positive link weights (i.e. bilateral trade flows): Figure 2 does exactly that by plotting the complementary cumulative probability distribution of trade flows in log-log scale, both for product-level transactions and for aggregate bilateral flows.[4]

[2]Detailed information on the issue are available upon request.

[3]In our dataset there are 17 countries that were formed after 1991 and represent therefore new entrants.

[4]Figure 2 refers to 1997 data, but other years display exactly the same behavior.

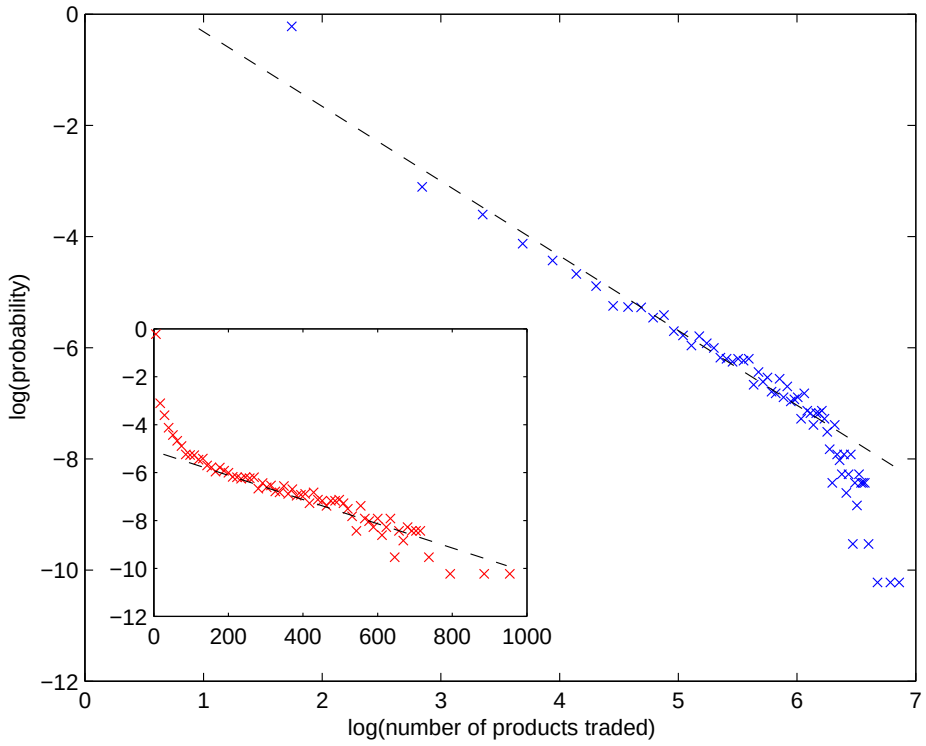

Figure 1: Distribution of the number of products traded – 1997. Double log scale (main plot) and semi-log scale (inset).

We observe that both distributions display the parabolic shape typical of the log-normal distribution, thus conforming to previous findings by Bhattacharya et al. (2008) and Fagiolo et al. (2009). Upon aggregation the power-law behavior of the upper tail become more apparent, as predicted by Growiec et al. (2008), but this departure from log-normality concerns a very small number of observations (0.16% in the case of commodities flows, 2.21% for aggregate flows).[5]

Figure 3 shows that the growth rates of aggregate trade flows display a distribution that fits the model's prediction. Goodness of fit tests, reported in Table 1, lead us to reject the hypotheses of a Gaussian or a Laplace distribution, whereas the distribution suggested in Fu et al. (2005) and a Generalized Exponential (GED)

[5]Estimations of the power-law fit have been obtained using the methodology developed by Clauset et al. (2009).

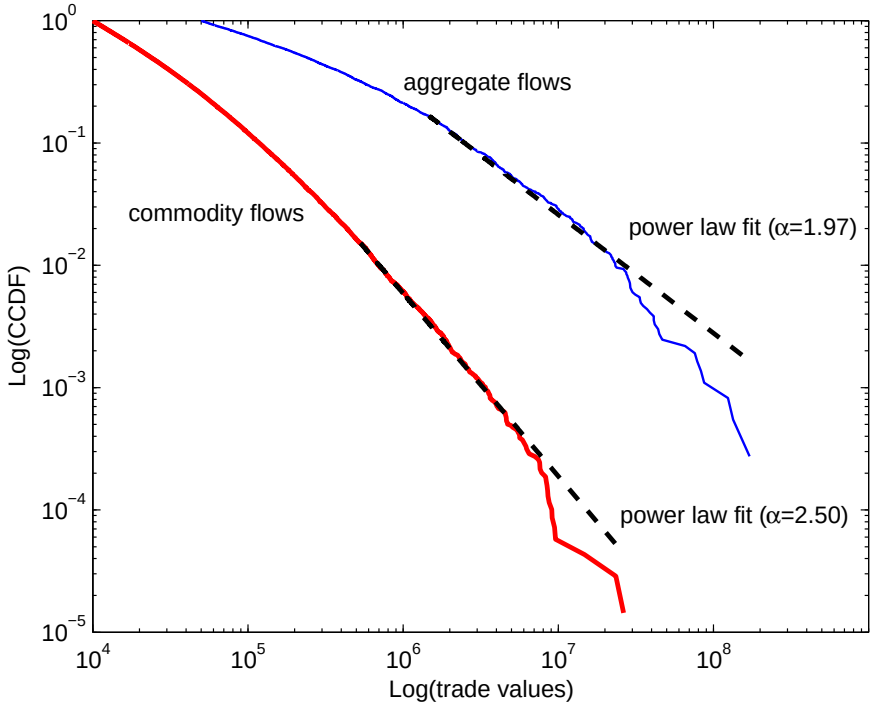

Figure 2: Positive link distribution – 1997. Complementary cumulative distribution of aggregate (blue) and commodity (red) flow and their power-law fits (dashed lines). Double logarithmic (base 10) scale.

perform much better in terms of Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests, making it difficult to discriminate among them. Hence, trade flows appear to follow a growth path similar to the one characterizing products, firms, industries, and country GDPs (Fu et al. 2005).

As discussed in Section 3, a simple model like the one presented here implies a negative relationship between the size and the variance of trade growth rates. Figure 4 looks at (the variance of) annual growth rates of aggregate trade flows, and their initial magnitude. As in Riccaboni et al. (2008), the variance of link weights growth appears to exhibit a crossover from (nearly) zero to -0.2, thus conforming to the model. This implies that the growth of the most intense trade flows are more volatile than expected based on the central limit theorem.

All in all, the main predictions of the model in terms of growth and size distribution of trade flows, number of commodities traded and size-variance relationship of trade flows are verified empirically. Thus we can conclude that a stochastic model that assumes a proportional growth of transactions as well as a multiplica-

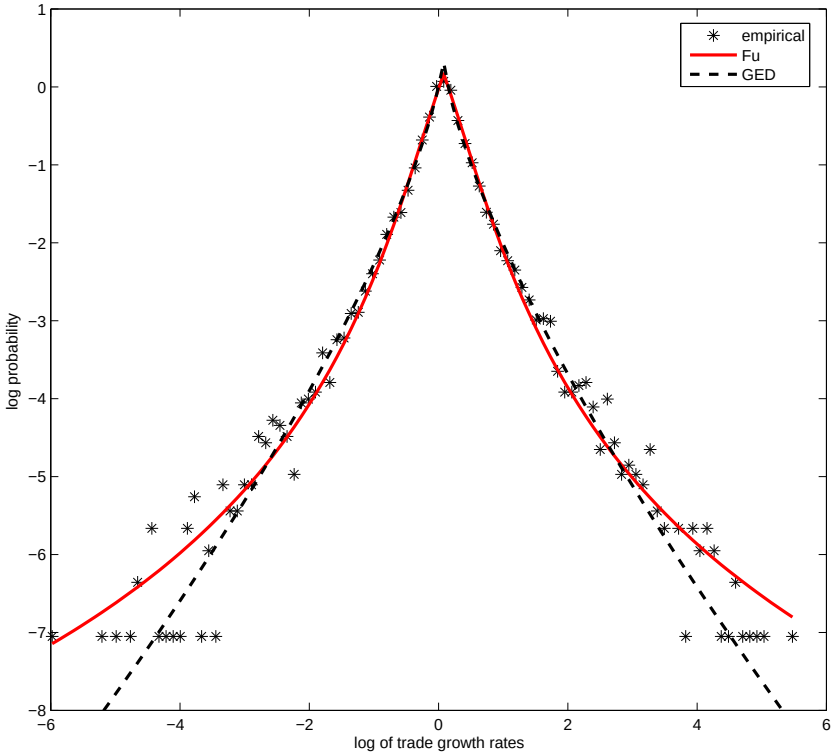

Figure 3: Distribution of the growth rates of aggregate trade flows

tive random growth of the value of each transactions can reproduce most of the observed structural features of the world trade web and should be taken as a valid stochastic benchmark to test the explanatory power of alternative theories of the evolution of international trade. In the next section we will compare the structure of random scale-free model networks generated according to our model and with the real world trade network.

5 Simulation and results

Based on the assumptions 1-3 of our model we can generate a set of random networks and fit them with real world data in order to verify the predictive capability of our theoretical framework and test alternative hypothesis about the evolution of the trade system. We will proceed in two steps. In the first stage, we generate the

Table 1: Goodness of fit tests, growth rates of aggregate trade flows

1992–1993

distribution	mode	par. 1	par. 2	par. 3	KS stat	AD stat
Fu	0.0219	0.4549			1.2391	0.0389
GED	0.0834	-0.0494	0.3083	0.7185	1.5507	0.0453
Gauss	0.0331	0.0014	0.9127		11.2891	2726.682
Laplace	0.0208	0.0035	0.5754		3.3476	0.3374

1999–2000

distribution	mode	par. 1	par. 2	par. 3	KS stat	AD stat
Fu	0.0798	0.3576			0.9325	0.0356
GED	0.0273	0.04421	0.2752	0.7214	1.2250	0.0297
Gauss	-0.0006	0.0826	0.8032		10.9902	298.8943
Laplace	0.0736	0.0014	0.5086		3.3075	0.1146

Pooled

distribution	mode	par. 1	par. 2	par. 3	KS stat	AD stat
Fu	0.0651	0.3658			0.8214	0.0477
GED	0.0191	0.0444	0.2899	0.7224	1.0915	0.0314
Gauss	0.0385	0.0333	0.8417		10.8305	117329
Laplace	0.0605	0.0040	0.5338		2.8414	1.4107

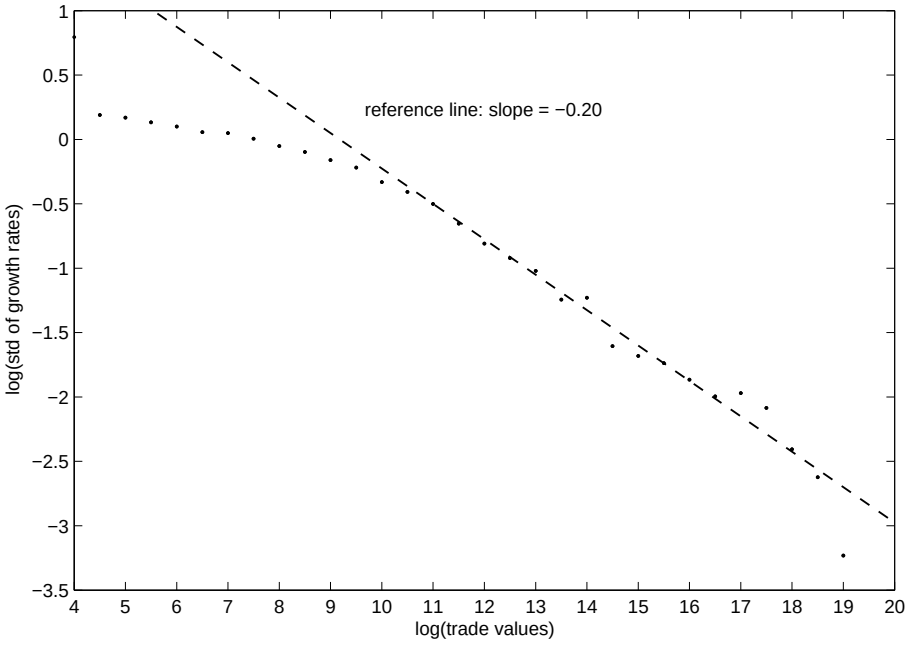

Figure 4: Size-variance relationship: aggregate trade

network of the number of transactions K. In the second stage, we assign the value of the transactions based on a random sampling of K values from a lognormal distribution whose parameters are obtain through a maximum likelihood fit of the real world distribution.

In the present context we model a system where at every instant t a trading opportunity arises, which represents the possibility to exchange one product with a partner country. We need to slightly modify the original setting in order to account for the fact that trade occurs between two parties, therefore each new opportunity needs to be assigned to two (different) players, an exporting and an importing country. So we end up with 4 parameters: a^{imp}, a^{exp}, b^{imp} and b^{exp}: the first two govern the entry of new destination and source countries while the other two control the amount of randomness in the allocation of opportunities among existing countries. In the baseline case we actually set $a^{imp} = a^{exp}$ and $b^{imp} = b^{exp}$ so that importing and exporting countries are treated symmetrically.

Interestingly, by tuning the two model parameters a and b it is possible to generate different types of outcomes in terms of the connectivity distribution of trade relations: i.e. the number of products exchanged by each country pair. In particular, with no entry ($a = 0$) and completely random allocation of opportunities ($b = 1$) one obtains the Erdös and Rnyi (1959) random graph characterized by a Poisson connectivity distribution, whereas allowing entry ($a > 0$) one moves towards an exponential distribution. Keeping a positive entry rate, but assigning opportunities according to a preferential attachment model ($b = 0$) the model reverts to the original Simon (1955) and Barabsi and Albert (1999) formulation, featuring a Pareto distribution for trading opportunities.[6] In the limit case in which entry of new players is ruled out ($a = 0$) then the connectivity distribution tends toward a Bose-Einstein geometric distribution.

We compare the structure of random scale-free model networks with the real world trade network in 1997.[7] In the first stage, we generate one million networks with a and b ranging from 0 to 1. We simulate random networks of 166 nodes (countries) and 1,079,398 links (number of different commodities traded by two countries). The number of commodities traded is taken as a proxy of the number of transactions. Next we select the random networks that better fit the real world pattern in terms of correlation, as measured by the Mantel r test, and connectivity distribution.

Figure 5 reports the value of the Mantel test for networks with $0 \geq b \leq 1$ and an entry rate a which implies the entry of 0 to 66 countries. The Mantel

[6]Many empirical studies have found that such kind of connectivity distribution characterizes a large number of (real-world) social, economic, or technological networks. This results explain the popularity of the (Barabsi and Albert 1999) model in network analysis.

[7]The structure of the network is highly stable other time. Results do not change substantially by comparing simulations with the structure of the real world network in different years.

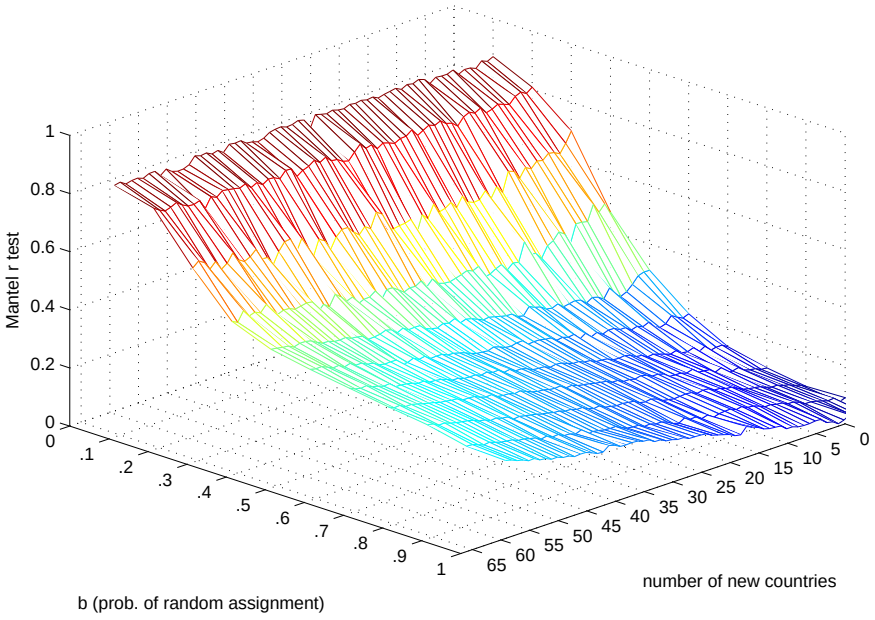

Figure 5: Mantel test comparing simulated and real K_ij

correlation statistics reach a peak of .88 (p¡.01) for poorly preferential attachment regimes ($b = 0$). However, the Mantel test does not discriminate among different entry regimes.

In order to do that, we can compare the connectivity distribution of simulated networks with the real world distribution of the number of traded commodities K by means of the Kolmogorov-Smirnov (KS) goodness of fit test. Figure 8 confirms that the best fit is obtained in the case of a purely preferential attachment allocation of trade opportunities ($b = 0$). However the KS tests provides additional information on the pattern of entry.

Figure 5 shows that the our model can better reproduce the connectivity distribution with and entry rate a that implies the entry of 14–18 countries. This closely corresponds to the empirically observed number of new countries. Thus we can conclude that a simple proportional growth model with mild entry can account for the distribution of the number of commodities traded by each pair of countries. An even better fit could be reached by simulating two different regimes for the importers and the exporters ($a^{imp} \neq a^{exp}$, $b^{imp} \neq b^{exp}$) but even in the simplified version the model provides an accurate description of the pattern of international transactions.

By introducing the value of the transactions we can show that the model gen-

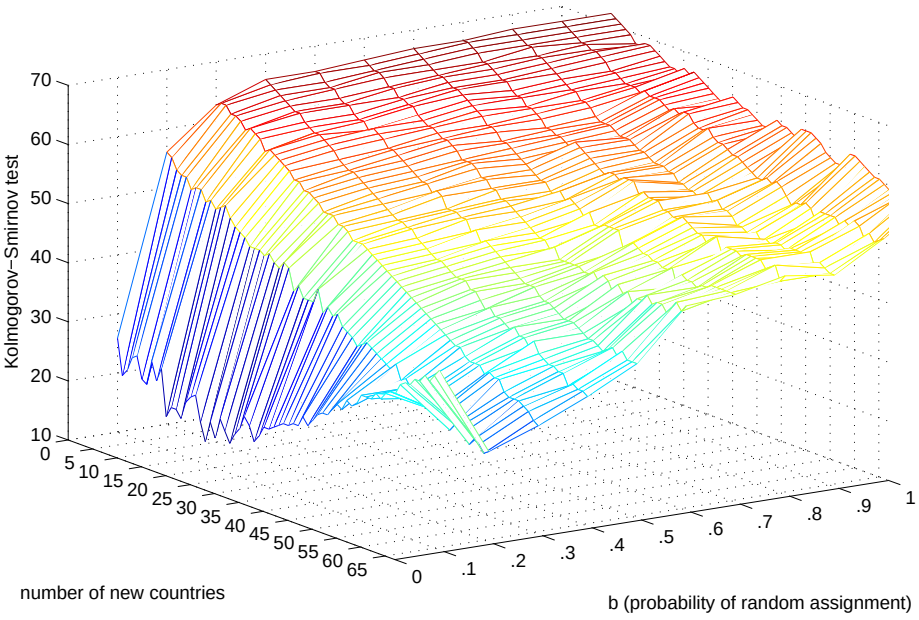

Figure 6: Kolmogorov-Smirnov goodness-of-fit test for different entry rates and probabilities of random assignment

erates the observed relationship between intensive and extensive margins of trade. Figure 8 depicts the relationship between total trade flows and the number of trade links maintained by each country. Empirically, we proxy the number of transactions by means of the number of products traded by each country.[8] Figure 8 displays the relationship at it emerges from 1997 trade data, and confirms that there exists a positive correlation between the two variables. The slope of the interpolating line (1.33) in double logarithmic scale reveals a positive relationship between the number of commodities and their average value. The curve displays and upward departure in the upper tail. This can be explained by noticing that the 4-digit SITC product classification that we used imposes a ceiling to the number of products a country can trade since there only around 1,300 4-digit categories (vertical dotted line). Apart from the upper decile of the distribution, the simulated version of the network shows exactly the same dependence among the size and the number of the transactions. This seems surprising by considering that the model assumes two independent growth processes for the number of transactions and their values. However, in should be noticed that the law of large numbers

[8]Note that $K_i \neq \sum_j K_{ij}$ since the same product can be exchanged with many different partners.

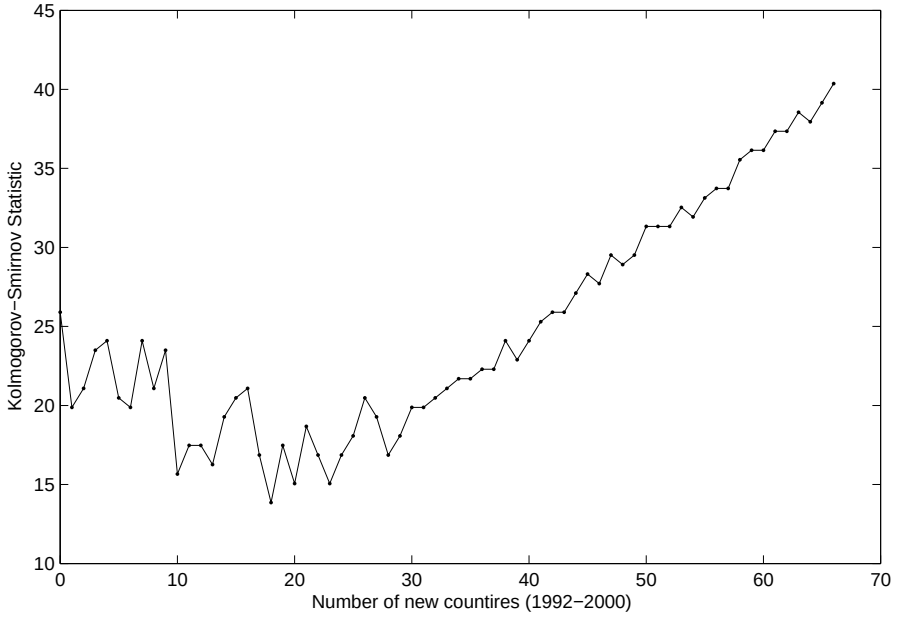

Figure 7: Kolmogorov-Smirnov goodness-of-fit test for different entry rates in a pure preferential attachment regime ($b = 0$)

does not work properly in case of skew distributions such as the lognormal. Given a random number of transactions with a finite expected value, if its values are repeatedly sampled from a lognormal, as the number of transactions increases, the average value of the transactions will tend to approach and stay close to the expected value (the average for the population). However this is true only for a sufficiently large number of transactions. The higher is the variance of the value of the transactions the larger is the number of transactions needed to approach the average value. Thus even the largest countries are far below the critical threshold. All in all, the relationship between intensive and extensive trade can be reproduced by means of a simple stochastic framework for the growth of trade flows. However, despite our simulated networks depicts a weak disassortative index, this is far below the real world value. Further work is needed to explain this stylized fact of the international trade network.

6 Discussion and conclusions

Using a simple model of proportionate growth and preferential attachment we are able to replicate the main structural properties of international trade flows. In

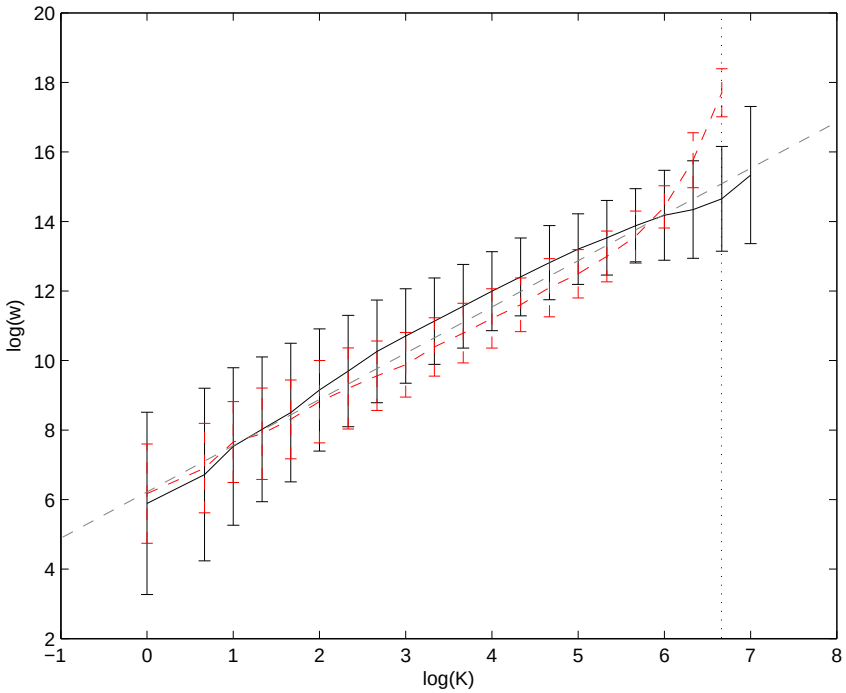

Figure 8: The relationship between the number of products traded and trade value. Double logarithmic scale. Simulated (back) and real-world (red) data, mean and one standard deviation in each direction.

particular, we provide an explanation to the power-law distribution of connectivity, as well as for the fat tails displayed by the distribution of the growth rates of trade flows. Additionally, the model matches the log-normal distribution of positive link weights (trade flows in the present context) and the negative relationship between the size of trade flows and the variance of their growth rates.

The contribution of the paper is twofold. First, we provide further evidence that a simple stochastic model is able to capture the dynamics of very different economic phenomena at different levels of aggregation, from business units, industrial sectors and country GDP (Fu et al. 2005) to international trade flows (as we show here). Second, we contribute to the discussion triggered by the Hummels and Klenow (2005) on the relative merits of various international trade models to adequately account for empirical facts. Indeed, we show that a simple model of proportionate growth is able to capture relevant features of the data, such as the fact that the extensive margin of trade account for a large fraction of the greater exports of large economies. We therefore provide a stochastic benchmark against which models can

be tested, in a vein that is similar to what is done by Sutton (2007) in the context of the debate on the persistence of industry leaders. Future international trade models should then focus on *departures* from this stochastic benchmark, which represent the result of the truly interesting economic forces at work.

For what concerns our next steps, further refinements of the model entail investigating its ability to match other topological properties of the networks such as the negative assortativity and hierarchical structure. For this we probably need to distinguish the mechanisms underlying export and import growth. More generally, while this paper does not emphasize the economic mechanisms through which countries are able to capture opportunities, this remains an important avenue for future research.

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